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# Coupled modes in parallel pillar microcavities: theory

G. Panzarini[a]

INFM - Dipartimento di Fisica "A. Volta", Università di Pavia, via Bassi 6, 27100 Pavia, Italy

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**Abstract.** The photonic modes of two identical, parallel microcavities are studied as a function of various parameters controlling the strength of the interaction between the isolated cavities. Coupling between the modes, which is determined by the overlap between the decaying e.m. field produced by one cavity in the region occupied by the second one, removes the degeneracy between the fundamental modes of the isolated structures. The energy splitting is found to depend strongly on the radius of the cavities, their distance, and the refractive index contrast between the core and the external medium.

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## 1 Introduction

It is well-known that the radiative properties of a material system are not an intrinsic and immutable characteristic of the electronic states of the emitter, but depend also on the electromagnetic (e.m.) field distribution and density of states at the emitter position. An extensive experimental and theoretical work has been developed in the last few years demonstrating the possibility to control the spontaneous and stimulated emission at optical frequencies by tailoring the photon states inside planar semiconductor microcavities (MCs) [1].

A further control of the emission properties has been achieved recently with the fabrication of laterally patterned semiconductor MCs. In these structures discretization of the spectrum of the optical modes in all three dimensions is achieved by combining vertical optical confinement by the distributed Bragg reflectors (DBRs) with the lateral optical confinement provided by the large refractive index contrast at the etched sidewalls. The cavity modes have been experimentally and theoretically studied both in photonic dots with rectangular cross section [2] and in pillar MCs (PMCs) [3]. Moreover, both the weak coupling regime [4,5], which manifests itself with a considerable increase of the spontaneous emission rate of the material system, and the formation of cavity polaritons in the strong coupling regime [6,7] have been demonstrated.

Interesting phenomena may also arise due to the coherent interactions between two cavities. This topic, which has been extensively studied by several groups in arrays of vertical-cavity surface emitting lasers [8], is a subject of both fundamental and practical interest: it provides an illustration of coherent interaction in semiconductor systems, as well as a means of transfer of e.m. power from one cavity to another. It has been shown [9] that coupling of the photon modes of two MCs with rectangular cross section *via* a narrow channel results in a splitting of the modes whose magnitude depends on the strength of the interaction, *i.e.* on the length and width of the channel.

Coherent interactions arise also if the two cavities are not connected through a channel, due to the leakage of the confined modes of the individual cavities in the surrounding material. In fact modes inside laterally patterned planar MCs are characterized by an amplitude in the external medium which, although rapidly decaying in the radial direction, does not vanish. The degree of leakage in the external region depends obviously on the mode, on the in-plane dimensions of the cavity, and on the indices of refraction of the core and of the cladding regions. If a second identical parallel cavity (MC B) is considered at a distance from the first one (MC A) where the amplitude of its decaying e.m. field is still appreciable, the guided modes of B are superimposed to the decaying modes produced by A in the core region of B, and *vice versa*. This is the source of the coupling between the cavities and of the splitting between otherwise degenerate modes.

The aim of this work is to study the coherent interaction between two PMCs, which is provided by the evanescent behavior of the fields in the cladding region. The splitting between the degenerate fundamental modes of the individual cavities is found to depend strongly on the radius of the PMCs, on their distance, and on the degree of confinement of the modes in the core region *via* the refractive index contrast between the internal and the external medium.

[a] e-mail: panzarini@pv.infn.it

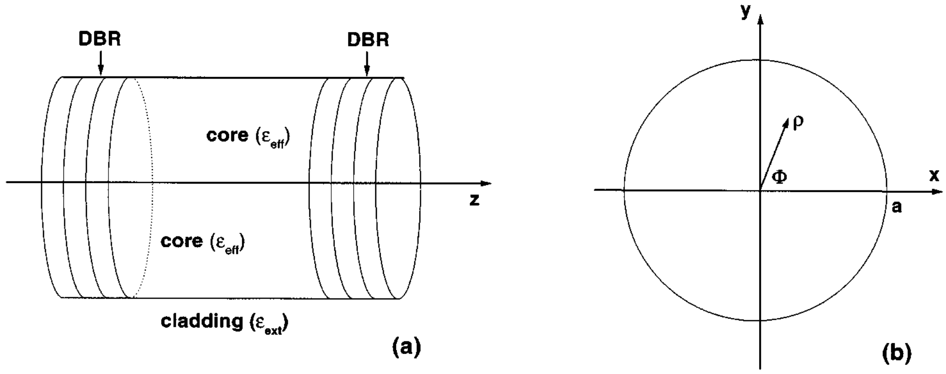


**Fig. 1.** (a) Schematic plot of a PMC. $\epsilon_{\rm eff}$ ($\epsilon_{\rm ext}$) is the dielectric function of the core (cladding) region. (b) Cross section of a PMC; the Cartesian and cylindrical coordinate systems are also displayed.

## 2 Model and formalism

### 2.1 Single cavity

A schematic representation of the structure and of its cross section is given in Figure 1, together with the employed Cartesian and cylindrical coordinate system.

The core region is formed by a central semiconductor layer with a circular cross section of radius $a$, extending between two highly reflecting DBRs which confine radiation in the vertical ($z$) direction. The dielectric constant of the external region $\epsilon_{\rm ext}$ is assumed to be smaller than that of the core in order for confined optical modes to exist inside the PMC. Penetration of the fields in the DBRs is accounted for by evaluating the effective length of the cavity $L_{\rm eff}$ and the effective dielectric constant of the core region $\epsilon_{\rm eff}$, a task which is easily performed analytically once the thicknesses and indices of refraction of each layer are known [11].

As further discussed in the following, the coherent interaction between the two cavities produces coupled modes with a splitting which is at most of a few meV; thus the effect can be detected only if the quality factor of the cavity is $\geq$ 500. At present this result has been achieved only with GaAs PMCs having radii $\geq 0.3\,\mu$m, since at smaller radii scattering by the roughness at the sidewalls considerably reduces the photon lifetime. It can be shown that at these radii -which is also the case investigated in the present work- vertical confinement of the photon modes dominates over lateral dielectric confinement, and the fields may be written as the product between two terms depending only on the longitudinal ($z$) or on the transverse ($\rho, \Phi$) coordinates, respectively.

The mode energies are evaluated according to a self-consistent procedure which is extensively discussed in reference [10]. This method provides two sets of discrete EH and HE modes for the PMC, a nomenclature which is reminiscent of that usually employed for cylindrical waveguides [12]. For a fixed polarization $\alpha$ ($\alpha$ = EH, HE) the frequency of the mode $\omega_{ln}^{(\alpha)}$ is labelled by two quantum numbers $(l, n)$: $l = 0, \pm 1, \pm 2...$ is the $z$ component of the total angular momentum, and $n = 1, 2, 3...$ denotes the successive roots of the characteristic equation for a fixed azimuthal quantum number $l$ and polarization $\alpha$. Modes with total angular momentum $l$ and $-l$ are degenerate and correspond to fields with positive and negative sense of rotation about the $z$ axis, respectively. The field $\mathbf{e}_{ln}^{(\alpha)}$ corresponding to the eigenfrequency $\omega_{ln}^{(\alpha)}$ is characterized in general by a non-vanishing value of all three linearly polarized components; however, either the positive or the negative circular component dominates the observable in-plane intensity distribution of the mode, and is much stronger also than the longitudinal one. Only for $\mathrm{EH}_{0n}$ and $\mathrm{HE}_{0n}$ modes the two circular components have the same weight, and the corresponding fields have a pure TE and TM polarization respectively [10,12].

In this paper we will consider only the $\mathrm{HE}_{11}$ and $\mathrm{HE}_{-11}$ modes, which are the fundamental ones in PMCs, and which are well-separated in energy from all other $\mathrm{HE}_{ln}$ and $\mathrm{EH}_{ln}$ modes inside the structure for all radii considered in this work. The normalized electric field profile of these modes is approximately given by (upper sign: $l = 1$, lower sign: $l = -1$):

*Core region*

$$\tilde{\mathbf{e}}(\mathbf{r}, t) = \mathbf{e}(\rho) f(z) \mathrm{e}^{-\mathrm{i}\omega_m t}$$
$$\approx \frac{1}{\sqrt{2\pi}} \frac{1}{N} J_0(\beta_m \rho) f(z) \mathrm{e}^{-\mathrm{i}\omega_m t}\, \hat{\mathbf{u}}_{\pm}; \tag{1}$$

*Cladding region*

$$\tilde{\mathbf{e}}(\mathbf{r}, t) = \mathbf{e}(\rho) f(z) \mathrm{e}^{-\mathrm{i}\omega_m t}$$
$$\approx \frac{1}{\sqrt{2\pi}} \frac{1}{N} \left( \frac{\beta_m a}{q_m a} \frac{J_1(\beta_m a)}{K_1(q_m a)} \right) K_0(q_m \rho) f(z) \mathrm{e}^{-\mathrm{i}\omega_m t}\, \hat{\mathbf{u}}_{\pm}. \tag{2}$$

In the preceding expressions the quantum numbers $l = \pm 1$, $n = 1$ and the polarization index $\alpha$ = HE are understood; moreover $\omega_m = \omega_{11}^{(\mathrm{HE})}$. $\hat{\mathbf{u}}_{\pm}$ are the rotating unit vectors, which are expressed in terms of the Cartesian $(\hat{x}, \hat{y})$ and polar $(\hat{\rho}, \hat{\Phi})$ unit vectors as:

$$\hat{\mathbf{u}}_{\pm} = \frac{1}{\sqrt{2}} (\hat{x} \pm \mathrm{i}\hat{y}) = \frac{1}{\sqrt{2}} (\hat{\rho} \pm \mathrm{i}\hat{\Phi}) \mathrm{e}^{\pm \mathrm{i}\Phi}. \tag{3}$$

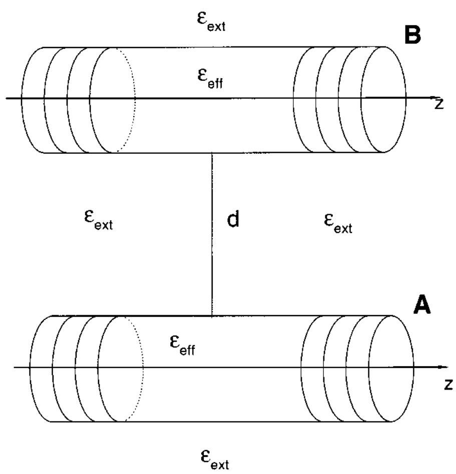


**Fig. 2.** Schematic representation of the coupled cavity structure formed by two identical, parallel PMCs with a minimum distance between their surfaces equal to $d$.

$J_l$ ($K_l$) is the Bessel function (modified Bessel function) of the first type of order $l$, $\beta_m = \sqrt{\epsilon_{\rm eff}\frac{\omega_m^2}{c^2} - \xi_m^2}$ ($q_m = \sqrt{\xi_m^2 - \epsilon_{\rm ext}\frac{\omega_m^2}{c^2}}$) is the transverse component of the mode wavevector in the core (cladding) region, $\xi_m$ is the longitudinal component of the wavevector, and $f(z)$ is a normalized function describing the $z$ dependence of the field, which is given by the usual transfer matrix approach for a planar MC. $N$ is the normalization factor, which is given by:

$$N = \sqrt{\int_0^a \mathrm{d}\rho\rho J_0^2(\beta_m\rho) + \left(\frac{\beta_m a}{q_m a}\frac{J_1(\beta_m a)}{K_1(q_m a)}\right)^2 \int_a^\infty \mathrm{d}\rho\rho K_0^2(q_{ln}\rho)}. \tag{4}$$

## 2.2 Coupled cavity structure

The structure, which is schematically represented in Figure 2, consists of two identical PMCs (denoted with A and B) of radius $a$ and effective dielectric function $\epsilon_{\rm eff}$: thus the dielectric function $\epsilon$ of the coupled structure is equal to $\epsilon_{\rm eff}$ in the core regions of A and B; elsewhere $\epsilon = \epsilon_{\rm ext}$.

The electric fields of the coupled cavity structure may be written as the sum of the individual unperturbed fields produced by A and B. Since the coupled cavity structure does not have cylindrical symmetry, the azimuthal quantum number $l$ is no more a good quantum number for its modes, and the fields should be expanded on a basis consisting of all modes with different $l$ of the isolated cavities. However, in order to describe the lowest energy modes, only the fundamental degenerate $\mathrm{HE}_{11}$ and $\mathrm{HE}_{-11}$ ones of the individual cavities can be considered since these modes are well-separated in energy from all other modes; thus

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{r},t) &= \mathbf{E}(\rho)f(z)\mathrm{e}^{-\mathrm{i}\omega_m t} \\ &= \sum_p \sum_{\mu=\{\mathrm{A,B}\}} A_p^{(\mu)}(t)\ \mathbf{e}_p^{(\mu)} f(z)\mathrm{e}^{-\mathrm{i}\omega_m t}. \end{aligned} \tag{5}$$

Here the superscript $(\mu)$ indicates that the quantity refers to the isolated PMC $\mu$ ($\mu = \mathrm{A,B}$). The subscript $p$ is a collective index representing the quantum numbers of the modes: thus $p = (1,1)$ or $p = (-1,1)$. $A_p^{(\mu)}(t)$ are complex coefficients. The $z$ dependence of the field is given by $f(z)$ and is the same both in the isolated cavity and in the coupled structure.

In order for $\tilde{\mathbf{E}}$ to be an eigenmode of the coupled cavity structure, $\tilde{\mathbf{E}}$ should satisfy the wave equation:

$$\nabla^2\ \tilde{\mathbf{E}} = \frac{\epsilon}{c^2}\frac{\partial^2\ \tilde{\mathbf{E}}}{\partial t^2}\cdot \tag{6}$$

Inserting expansion (5) in (6) the following relation is obtained:

$$\sum_p \sum_{\mu=\{\mathrm{A,B}\}} A_p^{(\mu)}(t)\hat{O}(\epsilon)\ \mathbf{e}_p^{(\mu)} =$$

$$\frac{\epsilon}{c^2}\sum_p \sum_{\mu=\{\mathrm{A,B}\}}\left(\frac{\partial^2 A_p^{(\mu)}}{\partial t^2} - 2\mathrm{i}\omega_m\frac{\partial A_p^{(\mu)}}{\partial t}\right)\mathbf{e}_p^{(\mu)}, \tag{7}$$

with

$$\hat{O}(\epsilon) = \nabla_{\mathrm{T}}^2 + \epsilon\frac{\omega_m^2}{c^2} - \xi_m^2, \tag{8}$$

$\nabla_{\mathrm{T}}^2$ being the transverse Laplacian operator.

When considering the operator $\hat{O}(\epsilon)$ acting on the unperturbed field $\mathbf{e}_p^{(\mu)}$, it is convenient to write $\epsilon$ in a form which emphasizes that PMC $\nu$ acts as a perturbing source on PMC $\mu$ if $\nu \neq \mu$, *i.e.* $\epsilon = \epsilon^{(\mu)} + (\epsilon - \epsilon^{(\mu)})$. Here $\epsilon^{(\mu)}$ denotes the dielectric function of the isolated PMC $\mu$: thus $\epsilon^{(\mu)} = \epsilon_{\rm eff}$ in the core region of cavity $\mu$; elsewhere $\epsilon^{(\mu)} = \epsilon_{\rm ext}$. Taking the scalar product in (7) with $\mathbf{e}_{\bar{p}}^{(\bar{\mu})\star}$ and integrating in the plane a system of four first-order linear differential equations in the expansion coefficients $A_p^{(\mu)}$ is finally obtained:

$$\frac{\mathrm{d}A_p^{(\mu)}}{\mathrm{d}t} = \mathrm{i}MA_p^{(\mu)} + \mathrm{i}\kappa A_p^{(\nu)}(t) \qquad (\nu \neq \mu), \tag{9}$$

with

$$M = \frac{\omega_m}{2\epsilon}\int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}y(\epsilon - \epsilon^{(\mu)})|\ \mathbf{e}_p^{(\mu)}|^2, \tag{10}$$

$$\kappa = \frac{\omega_m}{2\epsilon}\int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}y(\epsilon - \epsilon^{(\nu)})\ \mathbf{e}_p^{(\mu)\star}\cdot\ \mathbf{e}_p^{(\nu)}. \tag{11}$$

(9) has been derived under the assumption of slow time variation of the coefficients, *i.e.* $|\frac{\mathrm{d}^2 A_p^{(\mu)}}{\mathrm{d}t^2}| \ll |\omega_m\frac{\mathrm{d}A_p^{(\mu)}}{\mathrm{d}t}|$.

Note that the formalism developed here has strong analogies with the tight-binding approach in solid state physics, with $M$ and $\kappa$ representing the crystal field and the hopping term respectively.

Since the dielectric perturbation $(\epsilon - \epsilon^{(\mu)})$ couples only coefficients with the same quantum numbers, we can fix the index $p$, thus reducing the original system of four equations to only two coupled equations. Writing $A_p^{(\mu)}$ as

$$A_p^{(\mu)}(t) = v_p^{(\mu)} \mathrm{e}^{\mathrm{i}\Omega t} \tag{12}$$

with $v_p^{(\mu)}$ independent of time, the original differential problem is transformed into a standard algebraic one. The condition for the system to admit nontrivial solutions becomes:

$$\begin{pmatrix} \Omega - M & -\kappa \\ -\kappa & \Omega - M \end{pmatrix} \begin{pmatrix} v_p^{(\mathrm{A})} \\ v_p^{(\mathrm{B})} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{13}$$

The fundamental modes of the coupled cavity structure have thus energies $\omega_\pm = \tilde{\omega}_m \pm \kappa$ ($\tilde{\omega}_m = \omega_m - M$) and an electric field profile proportional to

$$\tilde{\mathbf{E}}_p^{(\pm)}(\mathbf{r}, t) \propto [\ \mathbf{e}_p^{(\mathrm{A})}(\mathbf{r}, t) \pm\ \mathbf{e}_p^{(\mathrm{B})}(\mathbf{r}, t)] f(z) \mathrm{e}^{-\mathrm{i}\tilde{\omega}_m t} \mathrm{e}^{\pm \mathrm{i}\kappa t}. \tag{14}$$

## 3 Results and discussion

Coupling between two parallel PMCs arises since A produces a field with an amplitude which -although small- is different from zero in the region occupied by B, and *vice versa*. The coupling removes the degeneracy between the fundamental eigenfrequencies of the individual cavities, and yields two doubly degenerate eigenmodes for the coupled structure with an energy separation $\Delta = 2\kappa$. Since the splitting depends on the strength of the interaction between the fields of the individual cavities, $\Delta$ can be varied simply by modifying the coupling between PMCs A and B.

The dielectric confinement of the cavity modes is related to the refractive index contrast at the lateral sidewalls. In [3,5,6] the PMCs have been fabricated by etching an epitaxially grown planar GaAs MC sample with GaAs/AlAs Bragg reflectors. The large refractive index contrast between GaAs and air yields e.m. waves which are very well-confined in the core region, as required in order to study phenomena such as the enhanced spontaneous emission rate of an internal light emitter, or the strong coupling regime between a quantum-well exciton and the cavity modes. In this work we are interested in the coupling between two parallel PMCs, which is increased on reducing the dielectric contrast since this process yields a larger penetration of the cavity modes in the external region. A particular favorable condition could be achieved, for example, by embedding two GaAs PMCs in a AlAs layer: total internal reflection takes still place at the lateral sidewalls, since AlAs has a smaller refractive index than GaAs; however the dielectric contrast is very small, thus allowing for an appreciable leakage of the cavity modes in the external medium.

Figure 3a shows the energy splitting $\Delta$ as a function of the refractive index of the external medium $n_{\mathrm{ext}} = \sqrt{\epsilon_{\mathrm{ext}}}$ for PMCs with radii $a = 300$ nm and a minimum distance between their surfaces (see Fig. 2) $d = 100$ nm (continuous line; left vertical scale) and $d = 300$ nm (dotted line; right vertical scale). Parameters have been chosen in order to describe the cavities employed by Gerard [3], and are the same as those of reference [10]; they correspond to a cavity with an effective length $L_{\mathrm{eff}} = 1096.13$ nm and an effective refractive index $n_{\mathrm{eff}} = 3.24$. It is apparent that the energy splitting is strongly enhanced on decreasing the dielectric contrast: at $d = 100$ nm, for example, it varies between $\Delta = 0.92$ meV for $n_{\mathrm{ext}} = 1$ and $\Delta = 6.45$ meV for $n_{\mathrm{ext}} = 2.9$.

In Figure 3b the energy splitting is studied as a function of the distance $d$ between the surfaces of A and B; we assume that the radius of the cavities is $a = 300$ nm and that the external medium has a refractive index $n_{\mathrm{ext}} = 1$ (continuous line; left vertical scale) and $n_{\mathrm{ext}} = 2.9$ (dotted line; right vertical scale). The exponential decay of $\Delta$ with increasing $d$ can be traced back to the exponential behavior of the field of the isolated cavity in the external medium (Eq. (2)): namely the radial dependence of the fields for large values of the argument may be approximated as:

$$K_0(q_m \rho) \approx \left(\frac{\pi}{2 q_m \rho}\right)^{1/2} \mathrm{e}^{-q_m \rho}. \tag{15}$$

Although $\Delta$ is obviously much larger for $n_{\mathrm{ext}} = 2.9$, the energy separation is appreciable also for $n_{\mathrm{ext}} = 1$ if $d <$ 200 nm; the possibility of resolving the two frequencies of the coupled structure in an optical experiment relies on the ability of fabricating PMCs with very high quality factors.

The last figure represents the energy splitting as a function of the radius $a$ when the distance between the surfaces of A and B is $d = 100$ nm and the external medium has an index of refraction $n_{\mathrm{ext}} = 1$ (continuous line; left vertical scale) and $n_{\mathrm{ext}} = 2.9$ (dotted line; right vertical scale). Comparison between Figures 3b and 3c shows that the splitting between the eigenmodes decreases at a faster rate if the separation between the centers of A and B is increased by keeping the radius $a$ fixed and increasing the distance $d$ between the surfaces, with respect to the case in which $d$ is kept constant and $a$ is increased. This is due to the fact that in the latter case the smaller amplitude of the field produced by one cavity in the region occupied by the second one with increasing the separation between their centers is partially compensated by the larger area in which the fields overlap.

In summary, the coupled modes of two identical cylindrical MCs have been studied in the present work. The overlap between the e.m. field produced by one cavity in the region occupied by the second one has been shown to remove the degeneracy between the modes of the isolated cavities. The splitting of the modes of the coupled structure depends on the strength of the interaction between the two cavities, which may be varied by changing

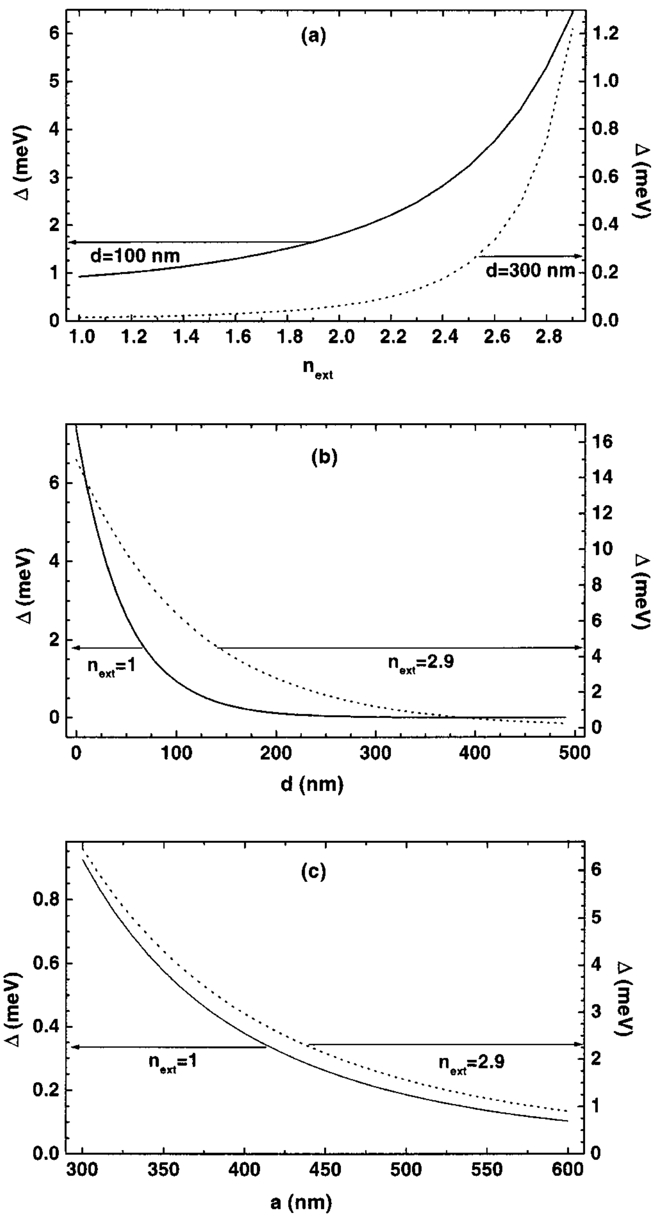


**Fig. 3.** Energy splitting between the coupled modes (a) as a function of the refractive index of the external medium. The two cavities have radii $a = 300$ nm and a minimum distance between their surfaces $d = 100$ nm (continuous line; left vertical scale) and $d = 300$ nm (dotted line; right vertical scale); (b) as a function of the minimum distance $d$ between A and B. The two cavities have radii $a = 300$ nm and are surrounded by an external medium with refractive index $n_{\text{ext}} = 1$ (continuous line; left vertical scale) and $n_{\text{ext}} = 2.9$ (dotted line; right vertical scale); (c) as a function of the radius $a$. The minimum distance between the surfaces of A and B is $d = 100$ nm; the external medium has a refractive index $n_{\text{ext}} = 1$ (continuous line; left vertical scale) and $n_{\text{ext}} = 2.9$ (dotted line; right vertical scale). Other parameters describing the structure are given in the text.

their radius and distance. The splitting, which is typically of the order of a few meV, is predicted to be experimentally detectable with state-of-the-art pillar MCs (which can now attain quality factors as large as $\sim 2\,000$ for radii $a \sim 0.5\,\mu$m). The possibly of observing the phenomenon rests however heavily on the ability of fabricating cavities with approximately the same radius. In order to overcome this technological difficulty, one can take advantage of the larger splitting which would be obtained on decreasing the refractive index contrast between the core and the external medium. This task is an exciting technological challenge, which would result in the demonstration of fundamental coherent phenomena in MCs, as well in interesting practical implementations.

## References


1. For reviews see *e.g. Confined Excitons and Photons: New Physics and Devices*, edited by E. Burstein, C. Weisbuch (Plenum, New York, 1995); *Microcavities and Photonic Bandgaps: Physics and Applications*, edited by C. Weisbuch, J. Rarity, NATO ASI Series E, Vol. **324** (Kluwer, Dordrecht, 1996).
2. J.P. Reithmaier, M. Röhner, H. Zull, F. Schäfer, A. Forchel, P.A. Knipp, T.L. Reinecke, Phys. Rev. Lett. **78**, 378 (1997).
3. J.M. Grard, D. Barrier, J.Y. Marzin, R. Kuszelewicz, L. Manin, E. Costard, V. Thierry-Mieg, T. Rivera, Appl. Phys. Lett. **69**, 449 (1996).
4. B. Ohnesorge, M. Bayer, A. Forchel, J.P. Reithmaier, N.A. Gippius, S.G. Tikhodeev, Phys. Rev. B **56**, R4367 (1997).
5. J.M. Grard, B. Sermage, B. Gayral, B. Legrand, E. Costard, V. Thierry-Mieg, Phys. Rev. Lett. **81**, 1110 (1998).
6. J. Bloch, F. Boeuf, J.M. Grard, B. Legrand, J.Y. Marzin, R. Planel, V. Thierry-Mieg, E. Costard, Physica E **2**, 915 (1998).
7. T. Gutbrod, M. Bayer, A. Forchel, J.P. Reithmaier, T.L. Reinecke, S. Rudin, P.A. Knipp, Phys. Rev. B **57**, 9950 (1998).
8. See *e.g.* P.L. Gourley, M.E. Warren, G.R. Hadley, G.A. Vawter, T.M. Brennan, B.E. Hammons, Appl. Phys. Lett. **58**, 890 (1991); M. Orenstein, E. Kapon, N.G. Stoffel, J.P. Harbison, L.T. Florez, J. Wullert, Appl. Phys. Lett. **58**, 804 (1991).
9. M. Bayer, T. Gutbrod, J.P. Reithmaier, A. Forchel, T.L. Reinecke, P.A. Knipp, A.A. Dremin, V.D. Kulakovskii, Phys. Rev. Lett. **81**, 2582 (1998).
10. G. Panzarini, L.C. Andreani, Phys. Rev. B **60**, 16799 (1999).
11. G. Panzarini, L.C. Andreani, A. Armitage, D. Baxter, M.S. Skolnick, V.N. Astratov, J.S. Roberts, A.V. Kavokin, M.R. Vladimirova, M.A. Kaliteevski Phys. Rev. B **59**, 5082 (1999).
12. N.S. Kapany, J.J. Burke, *Optical Waveguides* (Academic Press, New York and London, 1972).